

Evidence For A $1/r$ Long Range Gravity Force Term

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Abstract:

The Newton gravity inverse square law predicts velocities of stars decrease with increasing distance from a galaxy. Vera Rubin and others have observed velocities of stars and gas in the outer arms of galaxies to have nearly constant velocities. Addition of a $1/r$ force term to the Newton gravity equation provides a good match with observations. A derivation is presented in which existing gravitation fields produce an additional long range $1/r$ force component. This is not a MOND since the M/r^2 gravity field itself is not being modified. A hidden mass is thought to be inside existing mass and this causes the additional $1/r$ force.

Background:

Galaxies are rotating faster than predicted by Newton's gravity equation. Figure 1's expected force from Newton's equation is about half the actual force for $R=10,000$ light years, considering the centrifugal force is proportional to v^2 .

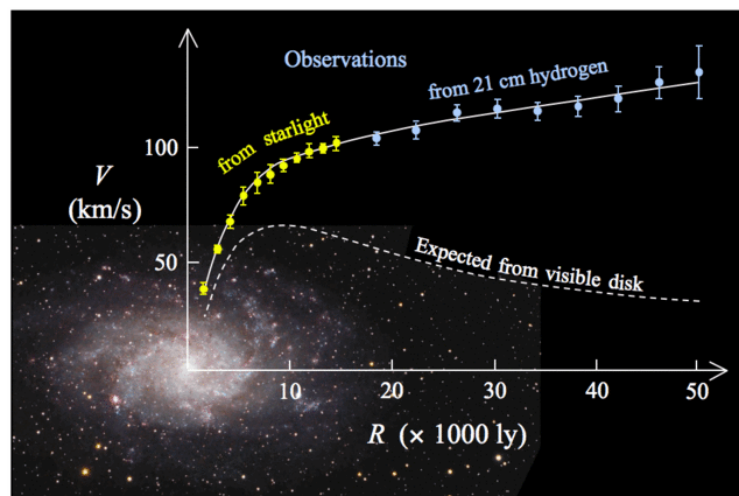


Fig. 1 – The M33 galaxy star velocities are made available to the public on [Wikipedia](#)¹. Newton's gravity equation shows expected velocities from the visible disk should taper off with increasing distance as shown in Figure 1. Actual velocities shown in the upper line from starlight and hydrogen in Figure 1 do not taper off. The assumption is that missing mass accounts for the wide discrepancy.

¹ Corbelli, E. & Salucci, P. (2000). "The extended rotation curve and the dark matter halo of M33". *Monthly Notices of the Royal Astronomical Society*. **311** (2): 441–447

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Fritz Zwicky² first noticed this discrepancy in the 1930s. Vera Rubin³ et al studied many galaxies and discovered the outer arms of galaxies move at nearly constant velocities. In 1932 J.H. Oort⁴ noted that stars just beyond the Oort Cloud are moving faster than expected. Oort assumed there must be hidden mass driving their greater than expected speeds.

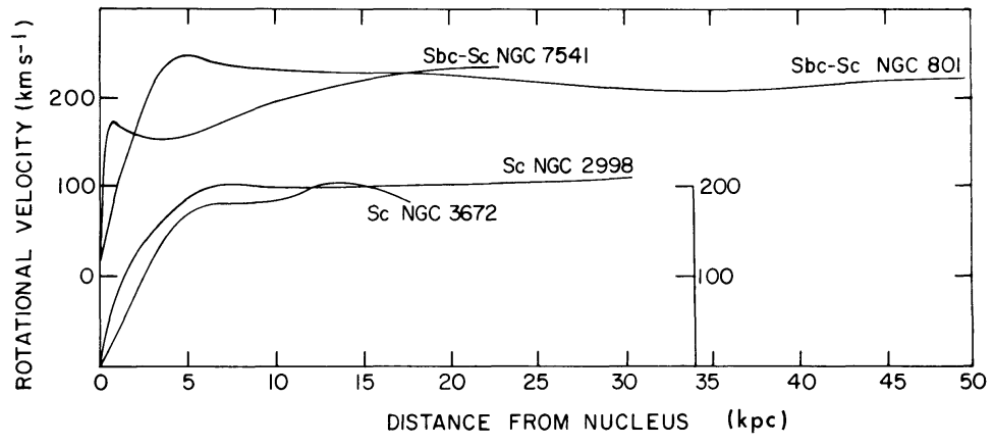


Fig. 2 – Is an example of a graph in reference 3 showing constancy of star velocities for distances well outside the main body of each galaxy observed.

A Dark Matter Review⁵ gives a history of the search for dark matter. John Moffat developed a modified (MOND) gravity theory⁶, however his MOND theory, which is based on General Relativity, did not match all observations. The Dark Matter Review paper does not mention any research in the testing of a $1/r$ force term. Dark matter remains the preferred theory to explain star movements.

Recently Martin Lo at JPL⁷ discovered $1/r$ works well in modeling the movements of stars in the outer arms of galaxies. Martin says the Cartwheel Galaxy is difficult to model using dark matter alone. He was able to model the Cartwheel Galaxy without dark matter when a $1/r$ force term is added.

² Zwicky, F. 1933, *Helv. Phys. Acta* 6, 110

³ Rubin, V.C., Ford, W.K. & Thonnard, N. 1978, *Astrophys. J.* 225, L107

⁴ J.H. Oort, 1932, *Bull. Astr. Inst. Netherlands*, 6, 249

⁵ V. Lukovic, P. Cabella, N. Vittorio, *DARK MATTER IN COSMOLOGY*, *International Journal of Modern Physics A*

⁶ J.W. Moffat, <https://arxiv.org/abs/gr-qc/0506021>

⁷ Martin W Lo, JPL, <https://arxiv.org/ftp/arxiv/papers/1305/1305.6847.pdf>

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For a period of time MOND was thought to be the cause of the Pioneer 10 anomaly, which was a de-acceleration of the satellite greater than predicted by Newton's gravity formula. A recent analysis shows the spacecraft is radiating heat which is causing an additional slowing down force⁸. A new burden on the $1/r$ force term is that it must now be consistent with Pioneer 10 observation of not being caused by a modified gravity while at the same time introducing an additional $1/r$ force term to explain galaxy rotations. I will show the $1/r$ term in this paper is much too weak to account for the Pioneer 10 anomaly.

The dark matter mass needed in our local star neighborhood is estimated to be as high as double the observed mass. This puts a burden on the missing mass theory to explain where this mass is since it is not yet observable. In the outer arms of galaxies the amount of dark matter needed is considerably more than the visible mass. The dark matter must also extend well beyond the visible stars to explain the speeds of hydrogen gas. Dark matter must also explain why the ratio of dark matter to visible matter increases in the outer arms of galaxies.

An intensive search for dark matter is currently underway. As time passes the likelihood of finding dark matter sources diminishes⁹. The $1/r$ force theory in this paper should merit attention as the search for missing mass continues.

As the search for dark matter continues, computer modelers may want to also test the effectiveness of adding a $1/r$ term. The $1/r$ force is of the form:

$$F = G \cdot m \cdot (U = \text{a new } M \text{ term or function})/r \quad (1)$$

In (1) the additional F gravity force from galaxy mass M balances the centrifugal force of star mass m to give nearly a constant velocity in the outer arms of a galaxy. The F must be a $1/r$ term to balance the $1/r$ centrifugal force. When $1/r \gg 1/r^2$ Newton's force, then Equation (2) $1/r$ force is in effect:

$$\text{centrifugal } mv^2/r = Gm(U/r) \text{ gravity} \quad (m \text{ is the star at location } r) \quad (2)$$

Then $v^2 = GU$ is a constant velocity for the very distant star. What is the U ?

⁸ Turyshev, S. G.; Toth, V. T.; Kinsella, G.; Lee, S.-C.; Lok, S. M.; Ellis, J. (2012). "Support for the Thermal Origin of the Pioneer Anomaly". *Physical Review Letters*. **108** (24): 241101.

⁹ Jeff Hecht, "Dark matter: What's the matter?", http://www.nature.com/nature/journal/v537/n7622_supp/full/537S194a.html

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Proposed Solution:

From Figure 1 we see that at some large distance R there is a crossover of forces between Newton's $1/r^2$ force and a new $1/r$ force term. At the distance R the two force components are equal, which suggests we can write this equation:

$$F = GMm/r^2 + GMm/(Rr) \quad (3)$$

The first term in (3) is Newton's equation and should agree with the lower curve of Figure 1. The sum of the two terms in (3) should agree with the upper curve in Figure 1. We observe in Figure 1 there is a distance R where the $1/r$ force seems to take over. It's very suggestive the force of $1/r$ is real, i.e. not ad hoc.

R is probably a function of M. Let $kR = \sqrt{M}$ and then $k = \sqrt{M}/R$. The crossover R will be inversely proportional to k for a given M. From Figure 1 the R appears to be $\sim 10,000$ light years or $\sim 10^{20}$ meters. M33's mass is $\sim 10^{40}$ kgm when dark matter is not included. Solving for $k = \sqrt{10^{40}}/10^{20} = 1$. This is a delightful surprise. Writing out the new modified Newton equation where k is the strength of the new $1/r$ field compared to Newton's gravity force for a given M and r:

$$F = GMm/r^2 + k * G\sqrt{M}m/r \quad (4)$$

Collecting the (M/r^2) terms shows the gravity field isn't actually modified:

$$F = G(M/r^2)m + k * G(M/r^2)^{1/2}m \quad (5)$$

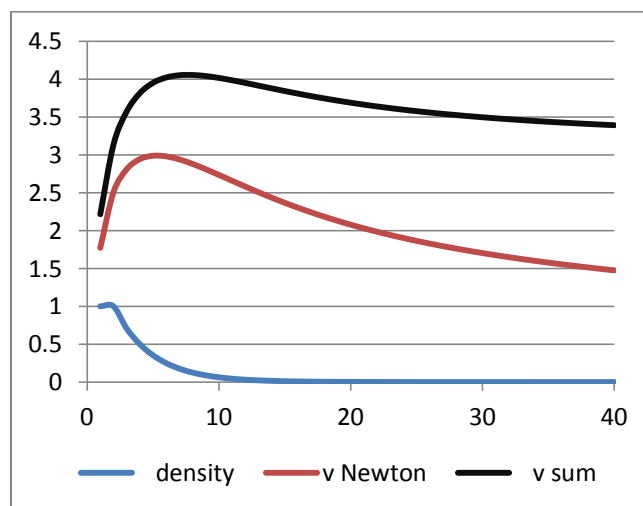


Fig. 3 – An example using Equation (5) was created to examine the shape of the velocity curves for Newton (red line) and additional $1/r$ force using $k=1$ (black line) for the blue line mass distribution. However, Figure 3 curves are incorrect.

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The GMm/r^2 is only Gaussian if the masses are in smooth constant density shells. This is not the shape of spiral galaxies. Both the red and black lines in Figure 3 are calculated incorrectly if the M is treated as a single point mass at $r=0$. Equation (5) must be integrated over all the individual masses in the galaxy and vector forces summed. The error is slight for $1/r^2$ but is very large for $1/r$ forces.

Exhibit 1 in the appendix is a computer program integrating small masses comprising a test galaxy M . M is a pancake shape of 40 concentric rings with r as 1 unit steps from 1 to 40. The disk is 1 unit thick. The first disk is $r=1$ so its area is π . The second disk is a ring from $r=1$ to $r=2$. The rings are broken up into 1 degree angle slices. The center of each slice is the center of the slice both by angle and radius. So the centers of the second ring are all at $r=1.5$ and angles .5, 1.5, 2.5, etc. degrees. This makes up a total of $360 \cdot 40 = 14400$ small masses. For each location r of mass m from 1 to 40 along the x axis, the force for both the $1/r^2$ term and the $1/r$ term is integrated over the entire set of 14400 masses.

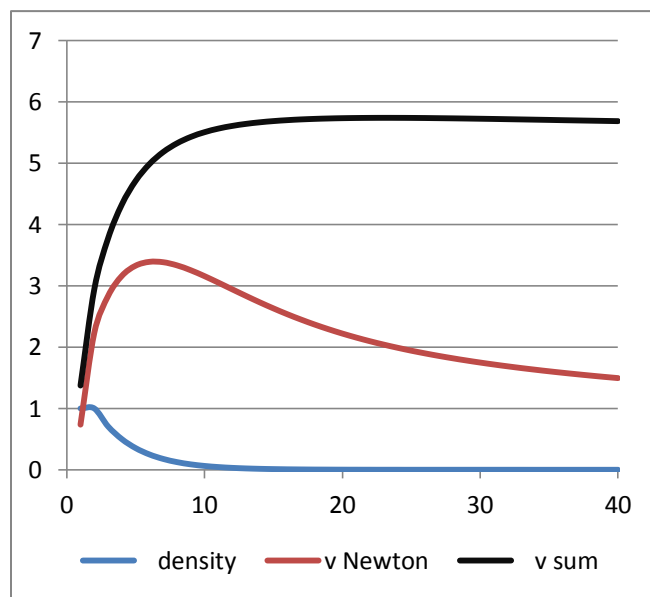


Fig. 4 – Equation (5) integrating the 14400 masses and setting $k=0.04$ shows velocities as a function of r using Newton's equation (red line) and Newton plus the $1/r$ force (black line). Setting $k=1$ in the computer program produced far too much $1/r$ force pushing the black line to large values not observed in Figure 1. Setting $k=0.04$ made the black and red lines have separation distances in Figure 4 approximately in agreement with Figure 1. The $1/r$ force must be a very weak force and this increases the R crossover distance to about 25 times greater than 10,000 light years. Integrating $1/r$ over all the masses is shown to be necessary.

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The $1/r$ Force on Pioneer 10:

Inserting the mass of the Sun into $R=\sqrt{M}/k$ and using the $k=0.04$ force multiplier adjustment gives a crossover $R = (2E30 \text{ kgm})^{.5} / .04 = 3.5E16$ meters. One light year is about $1E16$ meters, so the crossover distance is about 4 light years or the distance to our nearest star. When Pioneer 10 was at 80 AU or about $1E13$ meters from the Sun, the additional de-acceleration force on Pioneer 10 was about .001 per unit of Newton's force. The ratio of $1E13/4E16 = .00025$. The $1/r$ gravity force is much too small to be a factor causing Pioneer 10 to slow down.

The $1/r$ Force on Earth:

A calculation of the strength of the $1/r$ force on Earth is modeled in a computer program in Exhibit 2. The test includes 1000 meters of ice on top of a constant mass density for the rest of Earth. Two questions were asked. 1) how strong is the $1/r$ force compared to Newton's force and 2) is there any detectable surface force change that might be detectable? The experiment uses a 1 kgm weight. Exhibit 2 shows no near surface effect is detectable, and the $1/r$ force is only .000255 per unit of Newton's force on and below the surface.

Precession of Mercury:

A small computer program was written to study the precession of Mercury. The program shows that an application of formula for $k=0.016$ produces a precession of 42 arc seconds per Mercury years century so this is in the ball park. The program assumed Gaussian forces and this may be a source of error in the modeling of the $1/r$ term in the Mercury precession program.

The computer program was tested to insure that after 10000 Mercury years of orbiting that the Newton gravity formula produced an elliptical orbit with no precession due to numerical rounding. Then the $1/r$ term is added and a factor of $k=4.79$ was found to cause 360 degrees precession after 10000 Mercury years. The cause of the precession is that Mercury travels a bit faster at a greater distance from Mercury and this is cumulative. It would be the same nonlinearity as observed in stars orbiting a bit too fast in the outer arms of our galaxy.

So this $1/r$ term is a long range extra force. General Relativity predicts precession in a different way. The nearer Mercury is to the Sun, it effectively slows down. This can be modeled in the same program by modeling a k/r^3 term to produce the same kind of precession. But there is an oddity using the $1/r^3$ term. The k must be negative implying antigravity or a negative component to gravity. People don't think of GR as an antigravity force. But in this program it is.

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Conclusions:

The addition of a $1/r$ term to Newton's gravity equation is shown to be consistent with galaxy rotation velocities. Integration of the $1/r$ forces in Equation (5) shows the k must be lowered considerably from $k=1$. An approximate $k=0.04$ shows a good fit to M33 velocities. A good fit to Mercury's precession is $k=0.016$ for a Gaussian force assumption. The $1/r$ term is highly non Gaussian which requires summing up vector forces from a large number of clustered mass components. The $1/r$ force is very weak but also very long range.

The $1/r$ force crossover R extends far beyond Pioneer 10 to almost the distance of our nearest star. This is consistent with Pioneer 10 and local star observations in that the force is weak enough to not affect Pioneer 10 but strong enough to be included in the gravity forces of our local neighborhood stars.

The $1/r$ force on the surface of Earth is found to be only .000255 per unit of Newton's force. There is no surface variation seen in the computer output that could have been used to conduct an experiment.

Astronomy research should be testing the $1/r$ as an alternative to the existence of dark matter, especially now that observations are showing the dark matter assumption itself would produce illogical results, such as galaxies rotating around larger galaxies is inconsistent with the standard model.¹⁰

The $1/r$ term pulls (pun intended) astronomy into chaos theory models since the $1/r$ is a great long range attractor. The attractor might explain the barrel and spiral arms of our galaxy as well as the apparent existence of the so called planet 9 perturbations in our own solar system.

If there is a long range $1/r$ force we need to be able to explain its source. Obviously the source is matter. We need a description of atomic matter that causes the $1/r$ and $1/r^2$ forces. GR fails at very large distances and very small distances. The best candidate for what is causing the warping of space is not mass at the atomic level but is the strong electric fields. Mass just happens to be proportional. How does the electric field inside particles warp space in such a manner as to cause the $1/r$ and $1/r^2$ gravity forces between these particles?

¹⁰ <https://www.sciencedaily.com/releases/2018/02/180201142911.htm>
<https://astro.physik.unibas.ch/people/bruno-binggeli/oliver-mueller.html>

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Exhibit 1 Computer Program:

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* Program Gravity.for models Gaussian and the more complex integration of (M/r^2) and sqr(M/r^2).
* F1 IS 1/R FORCE F2 IS 1/R^2 FORCE
  REAL MASS,MASSR,DEN(40) ! MASSR IS TOTAL ENCLOSED MASS FOR DISTANCE R, MASS IS THE RING MASS
  INTEGER R
  PI=3.14159265
  WRITE(*,*) 'r      density      mass_M      M/r^2      sqr(M/r^2)      sum      (FORTRAN WRAPS AT COL 73)
&      v_Newton      v_sum'
  R=0.
  DENSITY=1.
  MASSR=0.
  DO 1 I=1,40
  IF(I.GT.2) DENSITY=DENSITY*.707 ! START DROPPING OFF THE GALAXY DENSITY FOR R>2
  MASS = DENSITY*PI*(I**2 - R**2) ! MASS OF THE RING
  R=I ! UPDATE R
  MASSR=MASSR + MASS ! TOTAL MASS OUT TO R
  DEN(R)=DENSITY ! REMEMBER THE RING DENSITIES FOR NEXT SECTION
  F2=MASSR/R**2 ! NEWTONS FORCE ASSUMING GAUSSIAN GM/R^2 HOLDS TRUE
  F1=SQRT(F2) ! SQUARE ROOT OF NEWTONS FORCE (DONT USE K SCALE FACTOR)
  SUM=F1+F2 ! SUM THE TWO FORCES
  VNEW=SQRT(F2*R) ! VELOCITY DUE TO NEWTONS FORCE
  VSUM=SQRT(SUM*R) ! VELOCITY DUE TO SQRT OF NEWTONS FORCE PLUS NEWTONS FORCE
  1 WRITE(*,FMT='(I2,F11.5,6F11.6)')R,DEN(R),MASSR,F2,F1,SUM,VNEW,VSUM
  WRITE(*,*)
  WRITE(*,*) 'SCALE FACTOR K ='
  READ(*,*) SFK
  WRITE(*,*) 'r      density      mass_M      M/r^2      sqr(M/r^2)      sum
&      v_Newton      v_sum'
* CALCULATE F1 AND F2 FORCES USING FULL INTEGRATIONS OF ALL NODES FOR 1 DEGREE PIE SECTIONS
  DEG1=2.*PI/360. ! theta angle = 1 degree increment
  DO 3 R=1,40 ! THERE ARE 40 RINGS, WE WILL TAKE THE MIDDLE OF EACH RING
  F1=0. ! ADD UP ALL THE F1 COMPONENT FORCES FOR EACH NEW LOCATION R ALONG THE X AXIS
  F2=0. ! LIKewise ADD UP ALL THE F2 COMPONENT FORCES FOR EACH NEW LOCATION R ALONG X
  MASSR=0. ! WE WILL CAPTURE THE MASS INSIDE R BUT HERE WE DO NOT NEED IT
  DO 2 I=40,1,-1 ! SWEEP OVER ALL NODES WITH MASS DEN(I) RADIUS I-.5 FROM THE X=0,Y=0 CENTER
  IF(I.LE.R) MASSR=MASSR+DEN(I)*2.*PI*(I-.5)
  MASS=DEN(I)*(I-.5)*DEG1 ! MASS=DENSITY*AREA OR MASS=DEN(I)*RADIUS*1DEGREE IN RADIANS
  DO 2 IANG=1,180 ! UPPER HALF PIES, EACH CENTER IS -.5 DEGREES, ANGLES ARE .5 TO 179.5 DEGREES
  X=(I-.5)*COS(PI*(IANg-.5)/180.)! X LOC OF MASS UPPER PLANE, I-.5 IS THE RADIUS OUT TO THE POINT
  Y=(I-.5)*SIN(PI*(IANg-.5)/180.)! Y LOC OF MASS UPPER PLANE, I-.5 IS THE RADIUS OUT TO THE POINT
  DIST=SQRT((R-X)**2+Y**2) ! DISTANCE FROM R TO THE ~CENTER OF THE RING SLICE OF MASS
  F1=F1+SFK*SQRT(MASS/DIST**2)*(R-X)/DIST ! CALCULATE THE F1 VECTOR FORCE COMPONENTS
  2 F2=F2+ (MASS/DIST**2)*(R-X)/DIST ! DO THE SAME CALCULATIONS FOR THE F2 NEWTON FORCE
  F1=F1+F1 ! ADD ON THE LOWER HALF OF F1 FORCES
  F2=F2+F2 ! ADD ON THE LOWER HALF OF F2 FORCES
  SUM=F1+F2 ! ADD THE TWO FORCE COMPONENTS
  VNEW=SQRT(F2*R) ! VELOCITY FROM NEWTONS EQUATION
  VSUM=SQRT(SUM*R) ! VELOCITY FROM NEWTONS PLUS NEW TERM ADDED
  3 WRITE(*,FMT='(I2,F11.5,6F11.6)')R,DEN(R),MASSR,F2,F1,SUM,VNEW,VSUM
  END

```


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Exhibit 1 first output report showing assumed Gaussian forces for 1/r and 1/r²:

| SCALE FACTOR K = 1 | | | | | | | |
|--------------------|---------|-----------|------------------|------------------------|----------|----------|----------|
| r | density | mass_M | M/r ² | sqr(M/r ²) | sum | v_Newton | v_sum |
| 1 | 1.00000 | 3.141593 | 3.141593 | 1.772454 | 4.914047 | 1.772454 | 2.216765 |
| 2 | 1.00000 | 12.566371 | 3.141593 | 1.772454 | 4.914047 | 2.506628 | 3.134979 |
| 3 | 0.70700 | 23.671902 | 2.630211 | 1.621793 | 4.252004 | 2.809027 | 3.571556 |
| 4 | 0.49985 | 34.664158 | 2.166510 | 1.471907 | 3.638417 | 2.943814 | 3.814927 |
| 5 | 0.35339 | 44.656116 | 1.786245 | 1.336505 | 3.122749 | 2.988515 | 3.951423 |
| 6 | 0.24985 | 53.290279 | 1.480286 | 1.216670 | 2.696955 | 2.980220 | 4.022652 |
| 7 | 0.17664 | 60.504517 | 1.234786 | 1.111209 | 2.345995 | 2.939983 | 4.052403 |
| 8 | 0.12489 | 66.389671 | 1.037339 | 1.018498 | 2.055837 | 2.880748 | 4.055452 |
| 9 | 0.08829 | 71.105247 | 0.877843 | 0.936933 | 1.814775 | 2.810797 | 4.041408 |
| 10 | 0.06242 | 74.831383 | 0.748314 | 0.865051 | 1.613365 | 2.735533 | 4.016672 |
| 11 | 0.04413 | 77.743065 | 0.642505 | 0.801564 | 1.444069 | 2.658487 | 3.985568 |
| 12 | 0.03120 | 79.997681 | 0.555539 | 0.745345 | 1.300885 | 2.581951 | 3.951027 |
| 13 | 0.02206 | 81.730301 | 0.483611 | 0.695422 | 1.179033 | 2.507378 | 3.915026 |
| 14 | 0.01560 | 83.053261 | 0.423741 | 0.650954 | 1.074695 | 2.435647 | 3.878883 |
| 15 | 0.01103 | 84.057877 | 0.373591 | 0.611221 | 0.984811 | 2.367247 | 3.843457 |
| 16 | 0.00780 | 84.817123 | 0.331317 | 0.575601 | 0.906918 | 2.302405 | 3.809290 |
| 17 | 0.00551 | 85.388542 | 0.295462 | 0.543564 | 0.839026 | 2.241173 | 3.776698 |
| 18 | 0.00390 | 85.817017 | 0.264867 | 0.514653 | 0.779520 | 2.183486 | 3.745846 |
| 19 | 0.00276 | 86.137260 | 0.238607 | 0.488475 | 0.727082 | 2.129211 | 3.716794 |
| 20 | 0.00195 | 86.375916 | 0.215940 | 0.464693 | 0.680633 | 2.078171 | 3.689534 |
| 21 | 0.00138 | 86.553291 | 0.196266 | 0.443019 | 0.639285 | 2.030169 | 3.664012 |
| 22 | 0.00097 | 86.684814 | 0.179101 | 0.423203 | 0.602304 | 1.984998 | 3.640149 |
| 23 | 0.00069 | 86.782127 | 0.164049 | 0.405030 | 0.569080 | 1.942456 | 3.617849 |
| 24 | 0.00049 | 86.853989 | 0.150788 | 0.388315 | 0.539103 | 1.902345 | 3.597008 |
| 25 | 0.00034 | 86.906952 | 0.139051 | 0.372896 | 0.511947 | 1.864478 | 3.577523 |
| 26 | 0.00024 | 86.945930 | 0.128618 | 0.358634 | 0.487252 | 1.828681 | 3.559291 |
| 27 | 0.00017 | 86.974564 | 0.119307 | 0.345408 | 0.464715 | 1.794793 | 3.542216 |
| 28 | 0.00012 | 86.995575 | 0.110964 | 0.333112 | 0.444076 | 1.762664 | 3.526206 |
| 29 | 0.00009 | 87.010971 | 0.103461 | 0.321654 | 0.425115 | 1.732160 | 3.511174 |
| 30 | 0.00006 | 87.022240 | 0.096691 | 0.310952 | 0.407644 | 1.703156 | 3.497043 |
| 31 | 0.00004 | 87.030472 | 0.090562 | 0.300936 | 0.391498 | 1.675540 | 3.483740 |
| 32 | 0.00003 | 87.036484 | 0.084997 | 0.291542 | 0.376538 | 1.649209 | 3.471199 |
| 33 | 0.00002 | 87.040871 | 0.079927 | 0.282714 | 0.362642 | 1.624070 | 3.459360 |
| 34 | 0.00002 | 87.044067 | 0.075298 | 0.274404 | 0.349702 | 1.600037 | 3.448168 |
| 35 | 0.00001 | 87.046394 | 0.071058 | 0.266568 | 0.337626 | 1.577035 | 3.437572 |
| 36 | 0.00001 | 87.048088 | 0.067167 | 0.259165 | 0.326332 | 1.554993 | 3.427530 |
| 37 | 0.00001 | 87.049316 | 0.063586 | 0.252163 | 0.315749 | 1.533846 | 3.417998 |
| 38 | 0.00000 | 87.050209 | 0.060284 | 0.245528 | 0.305812 | 1.513537 | 3.408939 |
| 39 | 0.00000 | 87.050858 | 0.057233 | 0.239233 | 0.296466 | 1.494012 | 3.400320 |
| 40 | 0.00000 | 87.051331 | 0.054407 | 0.233253 | 0.287660 | 1.475223 | 3.392110 |

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Exhibit 1 second output report showing non Gaussian 1/r and 1/r² for k=1:

SCALE FACTOR K = 1 Fig. 3

| r | density | mass_M | M/r ² | sqr(M/r ²) | sum | v_Newton | v_sum |
|----|---------|-----------|------------------|------------------------|-----------|----------|-----------|
| 1 | 1.00000 | 3.141593 | 0.541544 | 33.629967 | 34.171513 | 0.735897 | 5.845641 |
| 2 | 1.00000 | 12.566371 | 2.489649 | 45.939327 | 48.428978 | 2.231434 | 9.841644 |
| 3 | 0.70700 | 23.671902 | 2.675428 | 51.702793 | 54.378220 | 2.833070 | 12.772418 |
| 4 | 0.49985 | 34.664154 | 2.498672 | 54.503681 | 57.002354 | 3.161438 | 15.099980 |
| 5 | 0.35339 | 44.656116 | 2.215706 | 55.598091 | 57.813797 | 3.328443 | 17.002029 |
| 6 | 0.24985 | 53.290279 | 1.916541 | 55.623821 | 57.540363 | 3.391054 | 18.580694 |
| 7 | 0.17664 | 60.504517 | 1.636710 | 54.957851 | 56.594563 | 3.384815 | 19.903816 |
| 8 | 0.12489 | 66.389664 | 1.389111 | 53.841713 | 55.230824 | 3.333600 | 21.020147 |
| 9 | 0.08829 | 71.105247 | 1.176458 | 52.437222 | 53.613678 | 3.253940 | 21.966408 |
| 10 | 0.06242 | 74.831383 | 0.996924 | 50.856140 | 51.853065 | 3.157410 | 22.771269 |
| 11 | 0.04413 | 77.743065 | 0.846858 | 49.176067 | 50.022926 | 3.052120 | 23.457455 |
| 12 | 0.03120 | 79.997681 | 0.722109 | 47.452232 | 48.174343 | 2.943690 | 24.043547 |
| 13 | 0.02206 | 81.730301 | 0.618660 | 45.723167 | 46.341827 | 2.835945 | 24.544729 |
| 14 | 0.01560 | 83.053261 | 0.532894 | 44.016090 | 44.548985 | 2.731393 | 24.973701 |
| 15 | 0.01103 | 84.057877 | 0.461689 | 42.349541 | 42.811230 | 2.631603 | 25.341043 |
| 16 | 0.00780 | 84.817123 | 0.402422 | 40.735909 | 41.138329 | 2.537469 | 25.655666 |
| 17 | 0.00551 | 85.388542 | 0.352915 | 39.183414 | 39.536331 | 2.449399 | 25.925232 |
| 18 | 0.00390 | 85.817024 | 0.311385 | 37.696556 | 38.007942 | 2.367472 | 26.156126 |
| 19 | 0.00276 | 86.137268 | 0.276376 | 36.277611 | 36.553986 | 2.291538 | 26.353855 |
| 20 | 0.00195 | 86.375923 | 0.246713 | 34.927120 | 35.173832 | 2.221321 | 26.523134 |
| 21 | 0.00138 | 86.553299 | 0.221441 | 33.644493 | 33.865932 | 2.156447 | 26.668045 |
| 22 | 0.00097 | 86.684822 | 0.199790 | 32.428013 | 32.627804 | 2.096514 | 26.792007 |
| 23 | 0.00069 | 86.782135 | 0.181134 | 31.275415 | 31.456549 | 2.041098 | 26.897966 |
| 24 | 0.00049 | 86.853996 | 0.164970 | 30.184177 | 30.349148 | 1.989792 | 26.988508 |
| 25 | 0.00034 | 86.906960 | 0.150886 | 29.151512 | 29.302399 | 1.942205 | 27.065845 |
| 26 | 0.00024 | 86.945938 | 0.138550 | 28.174339 | 28.312889 | 1.897975 | 27.131809 |
| 27 | 0.00017 | 86.974571 | 0.127688 | 27.249723 | 27.377411 | 1.856765 | 27.188051 |
| 28 | 0.00012 | 86.995583 | 0.118076 | 26.374771 | 26.492847 | 1.818276 | 27.236000 |
| 29 | 0.00009 | 87.010971 | 0.109530 | 25.546463 | 25.655993 | 1.782240 | 27.276800 |
| 30 | 0.00006 | 87.022240 | 0.101898 | 24.762053 | 24.863951 | 1.748414 | 27.311508 |
| 31 | 0.00004 | 87.030472 | 0.095054 | 24.018824 | 24.113876 | 1.716583 | 27.340998 |
| 32 | 0.00003 | 87.036484 | 0.088890 | 23.314230 | 23.403120 | 1.686559 | 27.366034 |
| 33 | 0.00002 | 87.040871 | 0.083319 | 22.645800 | 22.729118 | 1.658174 | 27.387239 |
| 34 | 0.00002 | 87.044067 | 0.078267 | 22.011290 | 22.089556 | 1.631281 | 27.405199 |
| 35 | 0.00001 | 87.046394 | 0.073669 | 21.408558 | 21.482227 | 1.605747 | 27.420393 |
| 36 | 0.00001 | 87.048088 | 0.069473 | 20.835531 | 20.905005 | 1.581460 | 27.433195 |
| 37 | 0.00001 | 87.049324 | 0.065631 | 20.290413 | 20.356043 | 1.558314 | 27.444008 |
| 38 | 0.00000 | 87.050217 | 0.062105 | 19.771374 | 19.833479 | 1.536221 | 27.453091 |
| 39 | 0.00000 | 87.050865 | 0.058860 | 19.276844 | 19.335705 | 1.515105 | 27.460745 |
| 40 | 0.00000 | 87.051331 | 0.055870 | 18.805092 | 18.860962 | 1.494925 | 27.467043 |

Evidence For A 1/r Long Range Gravity Force Term

Exhibit 1 third output report showing non Gaussian 1/r and 1/r² for k=0.04:

SCALE FACTOR K = .04 Fig. 4

| r | density | mass_M | M/r ² | sqr(M/r ²) | sum | v_Newton | v_sum |
|----|---------|-----------|------------------|------------------------|----------|----------|----------|
| 1 | 1.00000 | 3.141593 | 0.541544 | 1.345198 | 1.886743 | 0.735897 | 1.373587 |
| 2 | 1.00000 | 12.566371 | 2.489649 | 1.837574 | 4.327223 | 2.231434 | 2.941844 |
| 3 | 0.70700 | 23.671902 | 2.675428 | 2.068114 | 4.743542 | 2.833070 | 3.772350 |
| 4 | 0.49985 | 34.664154 | 2.498672 | 2.180148 | 4.678820 | 3.161438 | 4.326116 |
| 5 | 0.35339 | 44.656116 | 2.215706 | 2.223926 | 4.439632 | 3.328443 | 4.711493 |
| 6 | 0.24985 | 53.290279 | 1.916541 | 2.224951 | 4.141491 | 3.391054 | 4.984872 |
| 7 | 0.17664 | 60.504517 | 1.636710 | 2.198313 | 3.835023 | 3.384815 | 5.181231 |
| 8 | 0.12489 | 66.389664 | 1.389111 | 2.153668 | 3.542779 | 3.333600 | 5.323742 |
| 9 | 0.08829 | 71.105247 | 1.176458 | 2.097492 | 3.273951 | 3.253940 | 5.428218 |
| 10 | 0.06242 | 74.831383 | 0.996924 | 2.034243 | 3.031166 | 3.157410 | 5.505603 |
| 11 | 0.04413 | 77.743065 | 0.846858 | 1.967046 | 2.813904 | 3.052120 | 5.563537 |
| 12 | 0.03120 | 79.997681 | 0.722109 | 1.898088 | 2.620197 | 2.943690 | 5.607349 |
| 13 | 0.02206 | 81.730301 | 0.618660 | 1.828929 | 2.447589 | 2.835945 | 5.640803 |
| 14 | 0.01560 | 83.053261 | 0.532894 | 1.760644 | 2.293537 | 2.731393 | 5.666526 |
| 15 | 0.01103 | 84.057877 | 0.461689 | 1.693980 | 2.155669 | 2.631603 | 5.686390 |
| 16 | 0.00780 | 84.817123 | 0.402422 | 1.629437 | 2.031858 | 2.537469 | 5.701731 |
| 17 | 0.00551 | 85.388542 | 0.352915 | 1.567335 | 1.920250 | 2.449399 | 5.713515 |
| 18 | 0.00390 | 85.817024 | 0.311385 | 1.507861 | 1.819246 | 2.367472 | 5.722449 |
| 19 | 0.00276 | 86.137268 | 0.276376 | 1.451106 | 1.727483 | 2.291538 | 5.729064 |
| 20 | 0.00195 | 86.375923 | 0.246713 | 1.397087 | 1.643800 | 2.221321 | 5.733760 |
| 21 | 0.00138 | 86.553299 | 0.221441 | 1.345780 | 1.567222 | 2.156447 | 5.736868 |
| 22 | 0.00097 | 86.684822 | 0.199790 | 1.297119 | 1.496909 | 2.096514 | 5.738640 |
| 23 | 0.00069 | 86.782135 | 0.181134 | 1.251016 | 1.432150 | 2.041098 | 5.739290 |
| 24 | 0.00049 | 86.853996 | 0.164970 | 1.207369 | 1.372339 | 1.989792 | 5.739001 |
| 25 | 0.00034 | 86.906960 | 0.150886 | 1.166060 | 1.316947 | 1.942205 | 5.737915 |
| 26 | 0.00024 | 86.945938 | 0.138550 | 1.126976 | 1.265526 | 1.897975 | 5.736173 |
| 27 | 0.00017 | 86.974571 | 0.127688 | 1.089991 | 1.217679 | 1.856765 | 5.733876 |
| 28 | 0.00012 | 86.995583 | 0.118076 | 1.054990 | 1.173066 | 1.818276 | 5.731131 |
| 29 | 0.00009 | 87.010971 | 0.109530 | 1.021860 | 1.131390 | 1.782240 | 5.728029 |
| 30 | 0.00006 | 87.022240 | 0.101898 | 0.990482 | 1.092381 | 1.748414 | 5.724632 |
| 31 | 0.00004 | 87.030472 | 0.095054 | 0.960753 | 1.055806 | 1.716583 | 5.721014 |
| 32 | 0.00003 | 87.036484 | 0.088890 | 0.932569 | 1.021459 | 1.686559 | 5.717227 |
| 33 | 0.00002 | 87.040871 | 0.083319 | 0.905831 | 0.989151 | 1.658174 | 5.713315 |
| 34 | 0.00002 | 87.044067 | 0.078267 | 0.880451 | 0.958718 | 1.631281 | 5.709327 |
| 35 | 0.00001 | 87.046394 | 0.073669 | 0.856341 | 0.930011 | 1.605747 | 5.705293 |
| 36 | 0.00001 | 87.048088 | 0.069473 | 0.833422 | 0.902894 | 1.581460 | 5.701245 |
| 37 | 0.00001 | 87.049324 | 0.065631 | 0.811616 | 0.877247 | 1.558314 | 5.697205 |
| 38 | 0.00000 | 87.050217 | 0.062105 | 0.790856 | 0.852961 | 1.536221 | 5.693199 |
| 39 | 0.00000 | 87.050865 | 0.058860 | 0.771072 | 0.829932 | 1.515105 | 5.689232 |
| 40 | 0.00000 | 87.051331 | 0.055870 | 0.752204 | 0.808074 | 1.494925 | 5.685328 |

Evidence For A 1/r Long Range Gravity Force Term

Exhibit 2 non Gaussian computer program for Earth's surface with 1000 m of ice:

```
* Program EarthGravity.for models the 1/r^2 and 1/r delta force of gravity into a 1000 M ice borehole.
  REAL*8 F1,F2,REARTH,R2,Z,D2,D,R
  REAL MASS,RATIO(1000)
  PI=3.14159265
  EMASS=6.E24
  DENSITY=920.
  GEARTH=9.8
  GNEWTON=6.67E-11
  REARTH=6371.D3
  VOLEARTH=(4./3.)*PI*REARTH**3
  EDENSITY=EMASS/VOLEARTH
  F1=0.D0
  F2=0.D0
  DO 1 Z=-REARTH+1,REARTH
  R2=REARTH**2-(Z-.5)**2
  R=DSQRT(R2)*.605
  MASS=PI*R2*EDENSITY
  IF(Z.LE.-REARTH+1) PRINT *,R,Z
  IF(Z.GE.REARTH) PRINT *,R,Z
  IF(Z.GE.REARTH-999) MASS=PI*R2*DENSITY! SWITCH TO STANDING ON ICE ~1000 M THICK
  D2=(REARTH-Z+.5)**2+R**2
  F1=F1+.04*(GNEWTON*SQRT(MASS/D2))*(R/DSQRT(D2))
  F2=F2+(GNEWTON*MASS/D2)*(R/DSQRT(D2)) ! THE VECTOR FORCE
  I=REARTH-Z
  IF(I.GE.1.AND.I.LE.1000) RATIO(I)=F1/F2
1 CONTINUE
  WRITE(*,(' F1 1/R FORCE =',F10.6/
& ' F2 1/R^2 FORCE =',F10.6/
& ' RATIO F1/F2 =',F10.6)')
& F1,F2,F1/F2
* ACTIVATE THIS SECTION TO LOOK AT THE TOP 1000 METERS OVER ICE, THERE IS NO SHORT RANGE EFFECT SEEN
* DO 2 I=1,1000
* 2 WRITE(*,('I4,F10.6)') I,RATIO(I)
  END
```

Exhibit 2 program output showing 1/r to 1/r² gravity is only .000255 pu, k=0.04:

```
1527.07083350103      -6370999.00000000
1527.07083350103      6371000.00000000
F1 1/R FORCE = 0.002505
F2 1/R^2 FORCE = 9.804832
RATIO F1/F2 = 0.000255
```