I need some help finding solutions to the differential equations below. Although the r distance from the center point in spherical coordinates is $0<r<\infty$, the majority of the change should be happening in the range around $r=10^{-15}$ meters. What is B and what is the relationship between ε and V?

Q = charge C = capacitance V = voltage

Q = CV and for a sphere C = 4πεr where ε is the permittivity of space r = the radius.

Q = 4πεrV but we allow no Q charge anywhere in the space to be created, therefore, letting <u>all derivatives be taken with respect to r</u>, we have:

$\partial Q = 0 = \partial(4\pi\varepsilon r V) = 0 = 4\pi\varepsilon V + 4\pi r\varepsilon\partial(V) + 4\pi r V\partial(\varepsilon)$ or

$$0 = \varepsilon V + r\varepsilon\partial(V) + rV\partial(\varepsilon) \qquad (1)$$

where ε and V are functions of r and V can have both DC and AC elements in solutions.

suppose $\varepsilon = \varepsilon_0 + BV^2$ (I chose V^2 to give symmetry to +/– V but this didn't happen)

$$0 = (\varepsilon_0 + BV^2)V + r(\varepsilon_0 + BV^2)\partial(V) + 2BrV^2V' \qquad (2)$$

This equation should have at least one non-trivial solution. The ∂(V) will be negative if V is positive, assuming r=0 is the center of the particle. However if V is negative the ∂(V) would be positive and V^3 would be negative. There does not appear to be enough symmetry between electrons and positrons in this equation.

Suppose $\varepsilon = \varepsilon_0 + B\partial(V)^2$ then

$$0 = \varepsilon V + r\varepsilon\partial(V) + rV\partial(\varepsilon) \qquad (3)$$

might be easier to solve. In general, the higher the power n of $\varepsilon = \varepsilon_0 + BV^n$ the weaker will be the gravity field which is ∂(ε). We can verify that by the gravity equation bending light around the sun. We also know the ∂(V) since that's just the electric field of an electron charge. There must be a unique solution to equation (1).

Gene Preston March 18, 2017

Solutions by Dale Gray: If I start with something like:

$$f(t)g(t) = C,$$

where C is a constant, and differentiate to get

$$f'(t)g(t) + f(t)g'(t) = 0,$$

I might call that a differential equation, and strictly speaking it is. But If I want to know a solution, the answer is trivial. A solution is $f(t)g(t) = C$.

When you start with $Q = 4\pi\epsilon r V$, and the assumptions that Q is a constant and that ϵ and V are functions of r, and differentiate both sides with respect to r, the "differential equation" you get has the solution

$$\epsilon r V = C,$$

where C is an arbitrary constant. This is basically what you started with. You have to start with some actual input to get something meaningful. Regarding the three differential equations you asked me about, a solution is

$$V = \frac{C}{\epsilon}\frac{1}{r},$$

where $\epsilon =$

$$\epsilon_0 + BV^2$$

, in the case of Eq.(2) and $\epsilon = \epsilon_0 + BV$, in the case of Eq.(3), and C is an arbitrary constant. By definition, a solution to a differential equation is a function that, when put into the equation, produces an identity. To get a unique solution you need to specify initial values. Substitute

$$V = \frac{C}{\epsilon_0 + BV^2}\frac{1}{r}$$

into your Eq.(2) to verify that it satisfies the equation. Do similarly for Eq.(3) and your corresponding choice of ϵ.

In other words, you have not provided enough information to determine anything more about V than what you started with, $\epsilon r V = C$. By the way, you have a math error in your Eq. (2). You forgot to use the chain rule in differentiating $\epsilon = \epsilon_0 + BV^2$ with respect to r. You should have gotten $\epsilon' = 2BVV'$. Your Eq.(2) should read

$$V'r(\epsilon_0 + 3BV^2) + BV^3 + \epsilon_0 V = 0.$$

However as I have said, your differential equation is irrelevant since you started with the solution to get the DE through differentiation. If you want V as a function of r, just solve the cubic equation

$$BV^3 + \epsilon_0 V = \frac{Q}{4\pi r}$$

for V. You can find the method of solving a cubic equation in a CRC manual.