Evidence For A 1/*r* Long Range Gravity Force From A New Interpretation Of Mass

Essay written for the Gravity Research Foundation 2017 Awards for Essays on Gravitation.

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Abstract:

The Newton gravity inverse square law predicts velocities of stars decrease with increasing distance from a galaxy. Vera Rubin and others have observed velocities of stars and gas in the outer arms of galaxies to have nearly constant velocities. Addition of a 1/*r* force term to the Newton gravity equation provides a good match with observations. A derivation is presented in which existing gravitation fields produce an additional long range 1/*r* force component. This is not a MOND since the M/r^2 gravity field itself is not being modified. A hidden mass is thought to be inside existing mass and this causes the additional 1/*r* force.

Background:

Galaxies are rotating faster than predicted by Newton's gravity equation. Figure 1's expected force from Newton's equation is about half the actual force for *R*=10,000 light years, considering the centrifugal force is proportional to v^2.

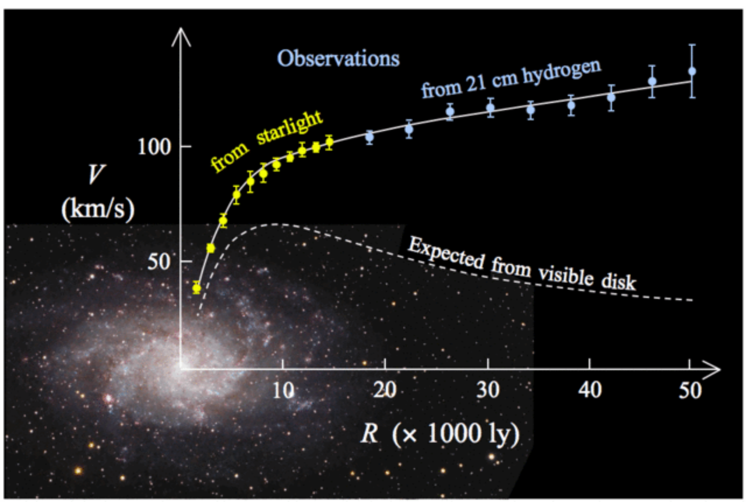

Fig. 1 – The M33 galaxy star velocities are made available to the public on Wikipedia[1]. Newton's gravity equation shows expected velocities from the visible disk should taper off with increasing distance as shown in Figure 1. Actual velocities shown in the upper line from starlight and hydrogen in Figure 1 do not taper off. The assumption is that missing mass accounts for the wide discrepancy.

[1] Corbelli, E. & Salucci, P. (2000). "The extended rotation curve and the dark matter halo of M33". *Monthly Notices of the Royal Astronomical Society*. **311** (2): 441–447

Fritz Zwicky[2] first noticed this discrepancy in the 1930s. Vera Rubin[3] et al studied many galaxies and discovered the outer arms of galaxies move at nearly constant velocities. In 1932 J.H. Oort[4] noted that stars just beyond the Oort Cloud are moving faster than expected. Oort assumed there must be hidden mass driving their greater than expected speeds.

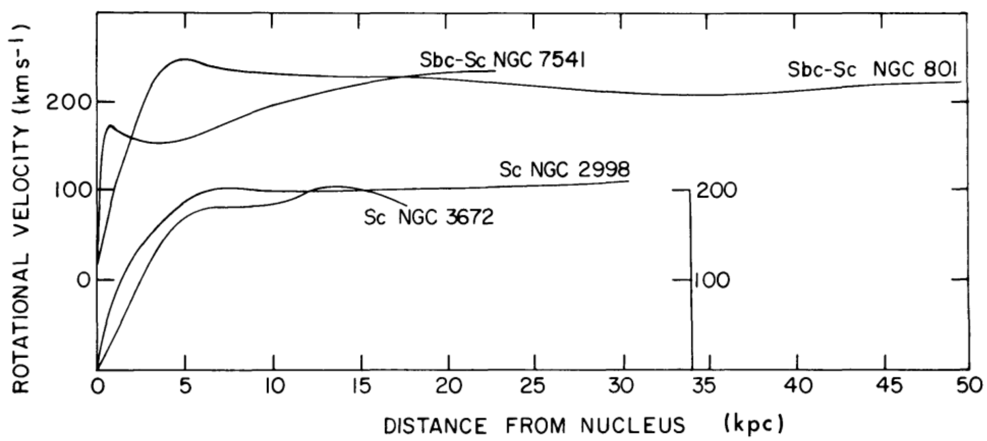

Fig. 2 – Is an example of a graph in reference 3 showing constancy of star velocities for distances well outside the main body of each galaxy observed.

A Dark Matter Review[5] gives a history of the search for dark matter. John Moffat developed a modified (MOND) gravity theory[6], however his MOND theory, which is based on General Relativity, did not match all observations. The Dark Matter Review paper does not mention any research in the testing of a 1/*r* force term. Dark matter remains the preferred theory to explain star movements.

Recently Martin Lo at JPL[7] discovered 1/*r* works well in modeling the movements of stars in the outer arms of galaxies. Martin says the Cartwheel Galaxy is difficult to model using dark matter alone. He was able to model the Cartwheel Galaxy without dark matter when a 1/*r* force term is added.

[2] Zwicky, F. 1933, Helv. Phys. Acta 6, 110
[3] Rubin,V.C., Ford, W.K. & Thonnard, N. 1978, Astrophys. J. 225, L107
[4] J.H. Oort, 1932, Bull. Astr. Inst. Netherlands, 6, 249
[5] V. Lukovic, P. Cabella, N. Vittorio, *DARK MATTER IN COSMOLOGY*, International Journal of Modern Physics A
[6] J.W. Moffat, https://arxiv.org/abs/gr-qc/0506021
[7] Martin W Lo, JPL, https://arxiv.org/ftp/arxiv/papers/1305/1305.6847.pdf

For a period of time MOND was thought to be the cause of the Pioneer 10 anomaly, which was a de-acceleration of the satellite greater than predicted by Newton's gravity formula. A recent analysis shows the spacecraft is radiating heat which is causing an additional slowing down force[8]. A new burden on the 1/*r* force term is that it must now be consistent with Pioneer 10 observation of not being caused by a modified gravity while at the same time introducing an additional 1/*r* force term to explain galaxy rotations. I will show the 1/*r* term in this paper is much too weak to account for the Pioneer 10 anomaly.

The dark matter mass needed in our local star neighborhood is estimated to be as high as double the observed mass. This puts a burden on the missing mass theory to explain where this mass is since it is not yet observable. In the outer arms of galaxies the amount of dark matter needed is considerably more than the visible mass. The dark matter must also extend well beyond the visible stars to explain the speeds of hydrogen gas. Dark matter must also explain why the ratio of dark matter to visible matter increases in the outer arms of galaxies.

An intensive search for dark matter is currently underway. As time passes the likelihood of finding dark matter sources diminishes[9]. The 1/*r* force theory in this paper should merit attention as the search for missing mass continues.

As the search for dark matter continues, computer modelers may want to also test the effectiveness of adding a 1/*r* term. The 1/*r* force is of the form:

$$F = \mathrm{G}\cdot\mathrm{m}\cdot(\mathrm{U} = \text{a new M term or function})/r \qquad (1)$$

In (1) the additional *F* gravity force from galaxy mass M balances the centrifugal force of star mass m to give nearly a constant velocity in the outer arms of a galaxy. The *F* must be a 1/*r* term to balance the 1/*r* centrifugal force. When $1/r \gg 1/r^2$ Newton's force, then Equation (2) 1/*r* force is in effect:

$$\text{centrifugal } \mathrm{m}v^2/r = \mathrm{Gm}(\mathrm{U}/r) \text{ gravity} \qquad (\text{m is the star at location } r) \qquad (2)$$

Then $v^2 = \mathrm{GU}$ is a constant velocity for the very distant star. What is the U?

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[8] Turyshev, S. G.; Toth, V. T.; Kinsella, G.; Lee, S.-C.; Lok, S. M.; Ellis, J. (2012). "Support for the Thermal Origin of the Pioneer Anomaly". *Physical Review Letters*. **108** (24): 241101.
[9] Jeff Hecht, "Dark matter: What's the matter?", http://www.nature.com/nature/journal/v537/n7622_supp/full/537S194a.html

Proposed Solution:

From Figure 1 we see that at some large distance R there is a crossover of forces between Newton's $1/r^2$ force and a new $1/r$ force term. At the distance R the two force components are equal, which suggests we can write this equation:

$$F = GMm/r^2 + GMm/(Rr) \tag{3}$$

The first term in (3) is Newton's equation and should agree with the lower curve of Figure 1. The sum of the two terms in (3) should agree with the upper curve in Figure 1. We observe in Figure 1 there is a distance R where the $1/r$ force seems to take over. It's very suggestive the force of $1/r$ is real, i.e. not ad hoc.

R is probably a function of M. Let $kR=\sqrt{M}$ and then $k=\sqrt{M}/R$. The crossover R will be inversely proportional to k for a given M. From Figure 1 the R appears to be ~10,000 light years or $\sim 10^{20}$ meters. M33's mass is $\sim 10^{40}$ kgm when dark matter is not included. Solving for $k=\sqrt{10^{40}}/10^{20}=1$. This is a delightful surprise. Writing out the new modified Newton equation where k is the strength of the new $1/r$ field compared to Newton's gravity force for a given M and r:

$$F = GMm/r^2 + k*G\sqrt{M}m/r \tag{4}$$

Collecting the (M/r^2) terms shows the gravity field isn't actually modified:

$$F = G(M/r^2)m + k*G(M/r^2)^{1/2}m \tag{5}$$

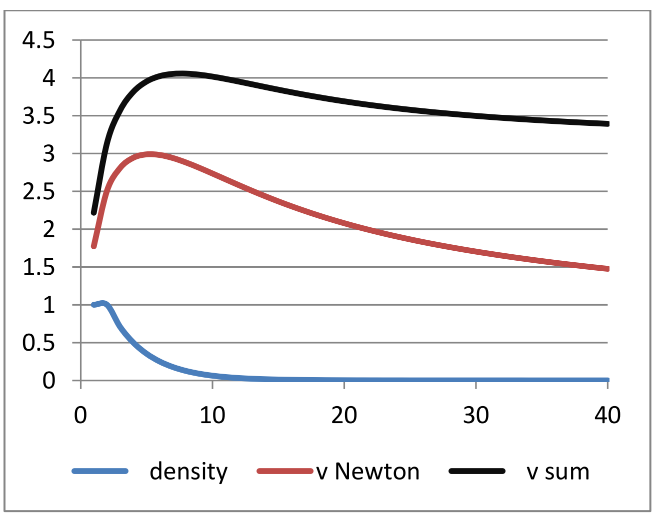

Fig. 3 – An example using Equation (5) was created to examine the shape of the velocity curves for Newton (red line) and additional $1/r$ force using k=1 (black line) for the blue line mass distribution. However, Figure 3 curves are incorrect.

The GMm/r^2 is only Gaussian if the masses are in smooth constant density shells. This is not the shape of spiral galaxies. Both the red and black lines in Figure 3 are calculated incorrectly if the M is treated as a single point mass at *r*=0. Equation (5) must be integrated over all the individual masses in the galaxy and vector forces summed. The error is slight for 1/r^2 but is very large for 1/*r* forces.

Exhibit 1 in the appendix is a computer program integrating small masses comprising a test galaxy M. M is a pancake shape of 40 concentric rings with *r* as 1 unit steps from 1 to 40. The disk is 1 unit thick. The first disk is *r*=1 so it's area is pi. The second disk is a ring from *r*=1 to *r*=2. The rings are broken up into 1 degree angle slices. The center of each slice is the center of the slice both by angle and radius. So the centers of the second ring are all at *r*=1.5 and angles .5, 1.5, 2.5, etc. degrees. This makes up a total of 360·40=14400 small masses. For each location *r* of mass m from 1 to 40 along the x axis, the force for both the 1/r^2 term and the 1/*r* term is integrated over the entire set of 14400 masses.

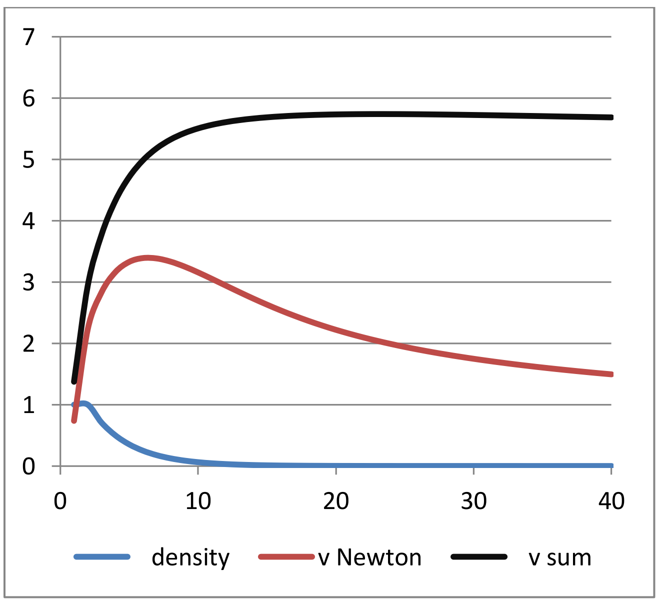

Fig. 4 – Equation (5) integrating the 14400 masses and setting k=0.04 shows velocities as a function of *r* using Newton's equation (red line) and Newton plus the 1/*r* force (black line). Setting k=1 in the computer program produced far too much 1/*r* force pushing the black line to large values not observed in Figure 1. Setting k=0.04 made the black and red lines have separation distances in Figure 4 approximately in agreement with Figure 1. The 1/*r* force must be a very weak force and this increases the R crossover distance to about 25 times greater than 10,000 light years. Integrating 1/*r* over all the masses is shown to be necessary.

Structure of mass speculation:

The source of the 1/*r* term force might be due to the structure of matter. Equation (5) suggests the gravity field itself is unchanged. The gravity potential -Gm/*r* remains unchanged. The 1/*r* in (5) suggests there are two types of m mass.

Matter is made up of individual particles. These particles, such as protons, neutrons, and electrons have natural vibration frequencies. In a classical mechanical sense anything that vibrates (or oscillates) is swapping energy between a potential stationary energy and a moving kinetic energy. In an electron the energy would be stored in a radial time varying electric field. The electron's electric field energy is swapping its energy content with another field containing the energy of motion. The two fields would be 90^{o} out of time phase.

The closest thing we have to describe the energy of radial motion in an electron is the renormalization virtual current. Renormalization theory needs to be expanded to include: 1) energy content of the electric and virtual currents, 2) particle vibration frequencies, and 3) the effect of an external gravity field on the potential and kinetic energies within the electron. It's very likely these two energies (masses) react differently to the external gravity field in Equation (5).

The 1/r Force on Pioneer 10:

Inserting the mass of the Sun into $R=\sqrt{M}/k$ and using the k=0.04 force multiplier adjustment gives a crossover R = (2E30 kgm)^.5 / .04 = 3.5E16 meters. One light year is about 1E16 meters, so the crossover distance is about 4 light years or the distance to our nearest star. When Pioneer 10 was at 80 AU or about 1E13 meters from the Sun, the additional de-acceleration force on Pioneer 10 was about .001 per unit of Newton's force. The ratio of 1E13/4E16 = .00025. The 1/*r* gravity force is much too small to be a factor causing Pioneer 10 to slow down.

The 1/r Force on Earth:

A calculation of the strength of the 1/*r* force on Earth is modeled in a computer program in Exhibit 2. The test includes 1000 meters of ice on top of a constant mass density for the rest of Earth. Two questions were asked. 1) how strong is the 1/*r* force compared to Newton's force and 2) is there any detectable surface force change that might be detectable? The experiment uses a 1 kgm weight. Exhibit 2 shows no near surface effect is detectable, and the 1/*r* force is only .000255 per unit of Newton's force on and below the surface.

Conclusions:

The addition of a 1/*r* term to Newton's gravity equation is shown to be consistent with galaxy rotation velocities. Integration of the 1/*r* forces in Equation (5) shows the k must be lowered considerably from k=1. An approximate k=0.04 shows a good fit to M33 velocities. The 1/*r* term is highly non Gaussian which requires summing up vector forces from a large number of clustered mass components. The 1/*r* force is very weak but also very long range.

The 1/*r* force crossover R extends far beyond Pioneer 10 to almost the distance of our nearest star. This is consistent with Pioneer 10 and local star observations in that the force is weak enough to not affect Pioneer 10 but strong enough to be included in the gravity forces of our local neighborhood stars.

The 1/*r* force on the surface of Earth is found to be only .000255 per unit of Newton's force. There is no surface variation seen in the computer output that could have been used to conduct an experiment.

Astronomy research should continue with use of Equation (5) for various k factors. Also, the research on matter itself should be expanded in an attempt to identify different kinds of mass within matter. The idea of mass consisting of potential and kinetic energy masses needs further theoretical and experimental verification. More specifically, the frequency of oscillation of a particle needs to be explained as virtual and kinetic energy swapping. Renormalization needs to be recalculated to include the effect of an external gravity field on the two types of internal energy (mass). This is a possible pathway for linking gravity with a more detailed description of matter. The 1/*r* and $1/r^2$ forces as functions of gravity should be derivable from more advanced models of individual particles' internal workings.

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Exhibit 1 Computer Program:

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* Program Gravity.for models Gaussian and the more complex integration of (M/r^2) and sqr(M/r^2).
* F1 IS 1/R FORCE  F2 IS 1/R^2 FORCE
      REAL MASS,MASSR,DEN(40)  ! MASSR IS TOTAL ENCLOSED MASS FOR DISTANCE R, MASS IS THE RING MASS
      INTEGER R
      PI=3.14159265
      WRITE(*,*) 'r    density    mass_M      M/r^2   sqr(M/r^2)     sum    (FORTRAN WRAPS AT COL 73)
     &     v_Newton     v_sum'
      R=0.
      DENSITY=1.
      MASSR=0.
      DO 1 I=1,40
      IF(I.GT.2) DENSITY=DENSITY*.707     ! START DROPPING OFF THE GALAXY DENSITY FOR R>2
      MASS = DENSITY*PI*(I**2 - R**2)     ! MASS OF THE RING
      R=I                                 ! UPDATE R
      MASSR=MASSR + MASS                  ! TOTAL MASS OUT TO R
      DEN(R)=DENSITY                      ! REMEMBER THE RING DENSITIES FOR NEXT SECTION
      F2=MASSR/R**2                       ! NEWTONS FORCE ASSUMING GAUSSIAN GM/R^2 HOLDS TRUE
      F1=SQRT(F2)                         ! SQUARE ROOT OF NEWTONS FORCE (DONT USE K SCALE FACTOR)
      SUM=F1+F2                           ! SUM THE TWO FORCES
      VNEW=SQRT(F2*R)                     ! VELOCITY DUE TO NEWTONS FORCE
      VSUM=SQRT(SUM*R)                    ! VELOCITY DUE TO SQRT OF NEWTONS FORCE PLUS NEWTONS FORCE
    1 WRITE(*,FMT='(I2,F11.5,6F11.6)')R,DEN(R),MASSR,F2,F1,SUM,VNEW,VSUM
      WRITE(*,*)
      WRITE(*,*) 'SCALE FACTOR K ='
      READ(*,*) SFK
      WRITE(*,*) 'r    density    mass_M      M/r^2   sqr(M/r^2)     sum
     &     v_Newton     v_sum'
* CALCULATE F1 AND F2 FORCES USING FULL INTEGRATIONS OF ALL NODES FOR 1 DEGREE PIE SECTIONS
      DEG1=2.*PI/360. ! theta angle = 1 degree increment
      DO 3 R=1,40     ! THERE ARE 40 RINGS, WE WILL TAKE THE MIDDLE OF EACH RING
      F1=0.           ! ADD UP ALL THE F1 COMPONENT FORCES FOR EACH NEW LOCATION R ALONG THE X AXIS
      F2=0.           ! LIKEWISE ADD UP ALL THE F2 COMPONENT FORCES FOR EACH NEW LOCATION R ALONG X
      MASSR=0.        ! WE WILL CAPTURE THE MASS INSIDE R BUT HERE WE DO NOT NEED IT
      DO 2 I=40,1,-1  ! SWEEP OVER ALL NODES WITH MASS DEN(I) RADIUS I-.5 FROM THE X=0,Y=0 CENTER
      IF(I.LE.R) MASSR=MASSR+DEN(I)*2.*PI*(I-.5)
      MASS=DEN(I)*(I-.5)*DEG1 ! MASS=DENSITY*AREA  OR  MASS=DEN(I)*RADIUS*1DEGREE IN RADIANS
      DO 2 IANG=1,180 ! UPPER HALF PIES, EACH CENTER IS -.5 DEGREES, ANGLES ARE .5 TO 179.5 DEGREES
      X=(I-.5)*COS(PI*(IANG-.5)/180.)! X LOC OF MASS UPPER PLANE, I-.5 IS THE RADIUS OUT TO THE POINT
      Y=(I-.5)*SIN(PI*(IANG-.5)/180.)! Y LOC OF MASS UPPER PLANE, I-.5 IS THE RADIUS OUT TO THE POINT
      DIST=SQRT((R-X)**2+Y**2)      ! DISTANCE FROM R TO THE ~CENTER OF THE RING SLICE OF MASS
      F1=F1+SFK*SQRT(MASS/DIST**2)*(R-X)/DIST    ! CALCULATE THE F1 VECTOR FORCE COMPONENTS
    2 F2=F2+        (MASS/DIST**2)*(R-X)/DIST    ! DO THE SAME CALCULATIONS FOR THE F2 NEWTON FORCE
      F1=F1+F1                                   ! ADD ON THE LOWER HALF OF F1 FORCES
      F2=F2+F2                                   ! ADD ON THE LOWER HALF OF F2 FORCES
      SUM=F1+F2                                  ! ADD THE TWO FORCE COMPONENTS
      VNEW=SQRT(F2*R)                            ! VELOCITY FROM NEWTONS EQUATION
      VSUM=SQRT(SUM*R)                           ! VELOCITY FROM NEWTONS PLUS NEW TERM ADDED
    3 WRITE(*,FMT='(I2,F11.5,6F11.6)')R,DEN(R),MASSR,F2,F1,SUM,VNEW,VSUM
      END
```

Exhibit 1 first output report showing assumed Gaussian forces for 1/*r* and 1/r^2:

SCALE FACTOR K = 1

r	density	mass_M	M/r^2	sqr(M/r^2)	sum	v_Newton	v_sum
1	1.00000	3.141593	3.141593	1.772454	4.914047	1.772454	2.216765
2	1.00000	12.566371	3.141593	1.772454	4.914047	2.506628	3.134979
3	0.70700	23.671902	2.630211	1.621793	4.252004	2.809027	3.571556
4	0.49985	34.664158	2.166510	1.471907	3.638417	2.943814	3.814927
5	0.35339	44.656116	1.786245	1.336505	3.122749	2.988515	3.951423
6	0.24985	53.290279	1.480286	1.216670	2.696955	2.980220	4.022652
7	0.17664	60.504517	1.234786	1.111209	2.345995	2.939983	4.052403
8	0.12489	66.389671	1.037339	1.018498	2.055837	2.880748	4.055452
9	0.08829	71.105247	0.877843	0.936933	1.814775	2.810797	4.041408
10	0.06242	74.831383	0.748314	0.865051	1.613365	2.735533	4.016672
11	0.04413	77.743065	0.642505	0.801564	1.444069	2.658487	3.985568
12	0.03120	79.997681	0.555539	0.745345	1.300885	2.581951	3.951027
13	0.02206	81.730301	0.483611	0.695422	1.179033	2.507378	3.915026
14	0.01560	83.053261	0.423741	0.650954	1.074695	2.435647	3.878883
15	0.01103	84.057877	0.373591	0.611221	0.984811	2.367247	3.843457
16	0.00780	84.817123	0.331317	0.575601	0.906918	2.302405	3.809290
17	0.00551	85.388542	0.295462	0.543564	0.839026	2.241173	3.776698
18	0.00390	85.817017	0.264867	0.514653	0.779520	2.183486	3.745846
19	0.00276	86.137260	0.238607	0.488475	0.727082	2.129211	3.716794
20	0.00195	86.375916	0.215940	0.464693	0.680633	2.078171	3.689534
21	0.00138	86.553291	0.196266	0.443019	0.639285	2.030169	3.664012
22	0.00097	86.684814	0.179101	0.423203	0.602304	1.984998	3.640149
23	0.00069	86.782127	0.164049	0.405030	0.569080	1.942456	3.617849
24	0.00049	86.853989	0.150788	0.388315	0.539103	1.902345	3.597008
25	0.00034	86.906952	0.139051	0.372896	0.511947	1.864478	3.577523
26	0.00024	86.945930	0.128618	0.358634	0.487252	1.828681	3.559291
27	0.00017	86.974564	0.119307	0.345408	0.464715	1.794793	3.542216
28	0.00012	86.995575	0.110964	0.333112	0.444076	1.762664	3.526206
29	0.00009	87.010971	0.103461	0.321654	0.425115	1.732160	3.511174
30	0.00006	87.022240	0.096691	0.310952	0.407644	1.703156	3.497043
31	0.00004	87.030472	0.090562	0.300936	0.391498	1.675540	3.483740
32	0.00003	87.036484	0.084997	0.291542	0.376538	1.649209	3.471199
33	0.00002	87.040871	0.079927	0.282714	0.362642	1.624070	3.459360
34	0.00002	87.044067	0.075298	0.274404	0.349702	1.600037	3.448168
35	0.00001	87.046394	0.071058	0.266568	0.337626	1.577035	3.437572
36	0.00001	87.048088	0.067167	0.259165	0.326332	1.554993	3.427530
37	0.00001	87.049316	0.063586	0.252163	0.315749	1.533846	3.417998
38	0.00000	87.050209	0.060284	0.245528	0.305812	1.513537	3.408939
39	0.00000	87.050858	0.057233	0.239233	0.296466	1.494012	3.400320
40	0.00000	87.051331	0.054407	0.233253	0.287660	1.475223	3.392110

Exhibit 1 second output report showing non Gaussian 1/*r* and $1/r^2$ for k=1:

SCALE FACTOR K = 1   Fig. 3

r	density	mass_M	M/r^2	sqr(M/r^2)	sum	v_Newton	v_sum
1	1.00000	3.141593	0.541544	33.629967	34.171513	0.735897	5.845641
2	1.00000	12.566371	2.489649	45.939327	48.428978	2.231434	9.841644
3	0.70700	23.671902	2.675428	51.702793	54.378220	2.833070	12.772418
4	0.49985	34.664154	2.498672	54.503681	57.002354	3.161438	15.099980
5	0.35339	44.656116	2.215706	55.598091	57.813797	3.328443	17.002029
6	0.24985	53.290279	1.916541	55.623821	57.540363	3.391054	18.580694
7	0.17664	60.504517	1.636710	54.957851	56.594563	3.384815	19.903816
8	0.12489	66.389664	1.389111	53.841713	55.230824	3.333600	21.020147
9	0.08829	71.105247	1.176458	52.437222	53.613678	3.253940	21.966408
10	0.06242	74.831383	0.996924	50.856140	51.853065	3.157410	22.771269
11	0.04413	77.743065	0.846858	49.176067	50.022926	3.052120	23.457455
12	0.03120	79.997681	0.722109	47.452232	48.174343	2.943690	24.043547
13	0.02206	81.730301	0.618660	45.723167	46.341827	2.835945	24.544729
14	0.01560	83.053261	0.532894	44.016090	44.548985	2.731393	24.973701
15	0.01103	84.057877	0.461689	42.349541	42.811230	2.631603	25.341043
16	0.00780	84.817123	0.402422	40.735909	41.138329	2.537469	25.655666
17	0.00551	85.388542	0.352915	39.183414	39.536331	2.449399	25.925232
18	0.00390	85.817024	0.311385	37.696556	38.007942	2.367472	26.156126
19	0.00276	86.137268	0.276376	36.277611	36.553986	2.291538	26.353855
20	0.00195	86.375923	0.246713	34.927120	35.173832	2.221321	26.523134
21	0.00138	86.553299	0.221441	33.644493	33.865932	2.156447	26.668045
22	0.00097	86.684822	0.199790	32.428013	32.627804	2.096514	26.792007
23	0.00069	86.782135	0.181134	31.275415	31.456549	2.041098	26.897966
24	0.00049	86.853996	0.164970	30.184177	30.349148	1.989792	26.988508
25	0.00034	86.906960	0.150886	29.151512	29.302399	1.942205	27.065845
26	0.00024	86.945938	0.138550	28.174339	28.312889	1.897975	27.131809
27	0.00017	86.974571	0.127688	27.249723	27.377411	1.856765	27.188051
28	0.00012	86.995583	0.118076	26.374771	26.492847	1.818276	27.236000
29	0.00009	87.010971	0.109530	25.546463	25.655993	1.782240	27.276800
30	0.00006	87.022240	0.101898	24.762053	24.863951	1.748414	27.311508
31	0.00004	87.030472	0.095054	24.018824	24.113876	1.716583	27.340998
32	0.00003	87.036484	0.088890	23.314230	23.403120	1.686559	27.366034
33	0.00002	87.040871	0.083319	22.645800	22.729118	1.658174	27.387239
34	0.00002	87.044067	0.078267	22.011290	22.089556	1.631281	27.405199
35	0.00001	87.046394	0.073669	21.408558	21.482227	1.605747	27.420393
36	0.00001	87.048088	0.069473	20.835531	20.905005	1.581460	27.433195
37	0.00001	87.049324	0.065631	20.290413	20.356043	1.558314	27.444008
38	0.00000	87.050217	0.062105	19.771374	19.833479	1.536221	27.453091
39	0.00000	87.050865	0.058860	19.276844	19.335705	1.515105	27.460745
40	0.00000	87.051331	0.055870	18.805092	18.860962	1.494925	27.467043

Exhibit 1 third output report showing non Gaussian 1/*r* and 1/r^2 for k=0.04:

SCALE FACTOR K = .04 Fig. 4

r	density	mass_M	M/r^2	sqr(M/r^2)	sum	v_Newton	v_sum
1	1.00000	3.141593	0.541544	1.345198	1.886743	0.735897	1.373587
2	1.00000	12.566371	2.489649	1.837574	4.327223	2.231434	2.941844
3	0.70700	23.671902	2.675428	2.068114	4.743542	2.833070	3.772350
4	0.49985	34.664154	2.498672	2.180148	4.678820	3.161438	4.326116
5	0.35339	44.656116	2.215706	2.223926	4.439632	3.328443	4.711493
6	0.24985	53.290279	1.916541	2.224951	4.141491	3.391054	4.984872
7	0.17664	60.504517	1.636710	2.198313	3.835023	3.384815	5.181231
8	0.12489	66.389664	1.389111	2.153668	3.542779	3.333600	5.323742
9	0.08829	71.105247	1.176458	2.097492	3.273951	3.253940	5.428218
10	0.06242	74.831383	0.996924	2.034243	3.031166	3.157410	5.505603
11	0.04413	77.743065	0.846858	1.967046	2.813904	3.052120	5.563537
12	0.03120	79.997681	0.722109	1.898088	2.620197	2.943690	5.607349
13	0.02206	81.730301	0.618660	1.828929	2.447589	2.835945	5.640803
14	0.01560	83.053261	0.532894	1.760644	2.293537	2.731393	5.666526
15	0.01103	84.057877	0.461689	1.693980	2.155669	2.631603	5.686390
16	0.00780	84.817123	0.402422	1.629437	2.031858	2.537469	5.701731
17	0.00551	85.388542	0.352915	1.567335	1.920250	2.449399	5.713515
18	0.00390	85.817024	0.311385	1.507861	1.819246	2.367472	5.722449
19	0.00276	86.137268	0.276376	1.451106	1.727483	2.291538	5.729064
20	0.00195	86.375923	0.246713	1.397087	1.643800	2.221321	5.733760
21	0.00138	86.553299	0.221441	1.345780	1.567222	2.156447	5.736868
22	0.00097	86.684822	0.199790	1.297119	1.496909	2.096514	5.738640
23	0.00069	86.782135	0.181134	1.251016	1.432150	2.041098	5.739290
24	0.00049	86.853996	0.164970	1.207369	1.372339	1.989792	5.739001
25	0.00034	86.906960	0.150886	1.166060	1.316947	1.942205	5.737915
26	0.00024	86.945938	0.138550	1.126976	1.265526	1.897975	5.736173
27	0.00017	86.974571	0.127688	1.089991	1.217679	1.856765	5.733876
28	0.00012	86.995583	0.118076	1.054990	1.173066	1.818276	5.731131
29	0.00009	87.010971	0.109530	1.021860	1.131390	1.782240	5.728029
30	0.00006	87.022240	0.101898	0.990482	1.092381	1.748414	5.724632
31	0.00004	87.030472	0.095054	0.960753	1.055806	1.716583	5.721014
32	0.00003	87.036484	0.088890	0.932569	1.021459	1.686559	5.717227
33	0.00002	87.040871	0.083319	0.905831	0.989151	1.658174	5.713315
34	0.00002	87.044067	0.078267	0.880451	0.958718	1.631281	5.709327
35	0.00001	87.046394	0.073669	0.856341	0.930011	1.605747	5.705293
36	0.00001	87.048088	0.069473	0.833422	0.902894	1.581460	5.701245
37	0.00001	87.049324	0.065631	0.811616	0.877247	1.558314	5.697205
38	0.00000	87.050217	0.062105	0.790856	0.852961	1.536221	5.693199
39	0.00000	87.050865	0.058860	0.771072	0.829932	1.515105	5.689232
40	0.00000	87.051331	0.055870	0.752204	0.808074	1.494925	5.685328

Exhibit 2 non Gaussian computer program for Earth's surface with 1000 m of ice:

```
* Program EarthGravity.for models the 1/r^2 and 1/r delta force of gravity into a 1000 M ice borehole.
      REAL*8 F1,F2,REARTH,R2,Z,D2,D,R
      REAL MASS,RATIO(1000)
      PI=3.14159265
      EMASS=6.E24                          ! kgm
      DENSITY=920.                         ! ICE kgm/cumeter
      GEARTH=9.8                           ! acceleration of gravity on Earth
      GNEWTON=6.67E-11
      REARTH=6371.D3
      VOLEARTH=(4./3.)*PI*REARTH**3
      EDENSITY=EMASS/VOLEARTH              ! gives 5539 kgm/cumeter
      F1=0.D0                              ! THE 1/R FORCE
      F2=0.D0                              ! NEWTON'S FORCE
      DO 1 Z=-REARTH+1,REARTH              ! WE ARE STANDING ON THE EARTH AT Z=REARTH
      R2=REARTH**2-(Z-.5)**2               ! GO .5 METER DEEPER FOR CENTER OF THE PANCAKE SECTION
      R=DSQRT(R2)*.605                     ! RADIUS DISTANCE TO RING TO GET 9.8 ACCEL FACTOR
      MASS=PI*R2*EDENSITY                  ! THIS IS PI R^2 TIMES DENSITY WHERE R2=R^2
      IF(Z.LE.-REARTH+1 ) PRINT *,R,Z      ! PRINT BOTTOM SLICE
      IF(Z.GE.REARTH    ) PRINT *,R,Z      ! PRINT TOP SLICE
      IF(Z.GE.REARTH-999) MASS=PI*R2*DENSITY! SWITCH TO STANDING ON ICE ~1000 M THICK
      D2=(REARTH-Z+.5)**2+R**2             ! DISTANCE SQUARED TO THE RING
      F1=F1+.04*(GNEWTON*SQRT(MASS/D2))*(R/DSQRT(D2))
      F2=F2+(GNEWTON*MASS/D2)*(R/DSQRT(D2)) ! THE VECTOR FORCE
      I=REARTH-Z
      IF(I.GE.1.AND.I.LE.1000) RATIO(I)=F1/F2
    1 CONTINUE
      WRITE(*,'(''   F1 1/R   FORCE ='',F10.6/
     &          ''   F2 1/R^2 FORCE ='',F10.6/
     &          ''   RATIO F1/F2    ='',F10.6)')
     &  F1,F2,F1/F2
* ACTIVATE THIS SECTION TO LOOK AT THE TOP 1000 METERS OVER ICE, THERE IS NO SHORT RANGE EFFECT SEEN
*      DO 2 I=1,1000
*    2 WRITE(*,'(I4,F10.6)') I,RATIO(I)
      END
```

Exhibit 2 program output showing 1/*r* to 1/r^2 gravity is only .000255 pu, k=0.04:

```
   1527.07083350103       -6370999.00000000
   1527.07083350103        6371000.00000000
   F1 1/R   FORCE =  0.002505
   F2 1/R^2 FORCE =  9.804832
   RATIO F1/F2    =  0.000255
```